Exam 1
Math 353
Summer Term I, 2021 **Name:** _____
Friday, May 28, 2021
Time Limit: 75 Minutes

This exam contains 7 pages (including this cover page) and 6 questions. The total number of points on this exam is 72. While this is a closed book, closed notes exam, you are allowed to use your one page review sheet, front and back, written on an 8.5 x 11 inch physical piece of paper in your own handwriting. While your review sheet must have been created with 20th century technology, you may take this exam on a tablet if you like, or on paper.

Show all of your work (on the backs of pages, if necessary) and circle your answers. If you do not completely solve a problem, explain what you do understand about it. If you would like to guess about a problem, clearly say so. This will maximize your partial credit. No collaboration on this exam is allowed. Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
Total:	72	

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1. (12 points) Consider the differential equation where

$$y' + \frac{2}{t}\, y = \frac{6\cos(2t)}{t^2}.$$

(a) Sketch the slope field for this differential equation.

(b) Solve this differential equation for $t \geq \pi$ with initial condition $y(\pi) = 5$.

2. (12 points) Consider the differential equation where

$$\frac{dy}{dt} = (y-1)(3-y)(5-y).$$

(a) Sketch the slope field for this differential equation.

(b) If $y(0) = 4$, what is the behavior of $y(t)$ as t goes to infinity?

(c) If $y(0) = 2$, what is the behavior of $y(t)$ as t goes to infinity?

3. (12 points) A swinging door (that can open both to the inside and the outside) has both a spring and a dampener on it to control how well the door shuts. Define the angle of the swinging door at time t to be $\theta(t)$, where $\theta = 0$ corresponds to a shut door, $\theta > 0$ is opening to the outside, and $\theta < 0$ is opening to the inside.

   Suppose that the dynamics of the swinging door are controlled by

   $$\theta''(t) = -4\theta(t) - 2k\theta'(t).$$

   In the above equation, the 4 represents the strength of the spring and the k represents the strength of the dampener (which resists motion in both directions).

   Find the general solution to the above differential equation when

   (a) k = 3

   (b) k = 2

   (c) k = 1

   (d) Which of the above solutions represents an "optimally damped" swinging door, defined to be the case where the door may never hit the door jam at $\theta = 0$ (if we insert one) but otherwise has a general solution that closes as quickly as possible (meaning that for very large t, $\theta(t)$ is as small as possible)?

4. (12 points) (a) Find the general solution to the equation $y'' - 5y' + 6y = 0$.

(b) Using the method of undetermined coefficients, find the general solution to the differential equation $y'' - 5y' + 6y = 2e^t$.

(c) Using variation of parameters, find the general solution to the differential equation $y'' - 5y' + 6y = 2e^t$.

5. (12 points) Use the Laplace transform to solve $y'' - 2y' + 5y = 0$ where $y(0) = 3$ and $y'(0) = -3$.

6. (12 points) The goal of this problem is to find the general solution to the differential equation $ty'' - 2y' + 9t^5 y = 0$, where y is a function of t.

   (a) Using the chain rule from single variable calculus, convert this to a differential equation where y is a function of x, where $x = t^3$.

   (b) Find the general solution to the differential equation for $y(x)$ you derived in part (a).

   (c) What is the general solution to the original differential equation for $y(t)$?