Exam 1
Math 353
Summer Term 1 **Name:** ____________________
May 27, 2022
Time Limit: 75 Minutes

This exam contains 6 pages (including this cover page) and 5 questions. The total number of points on this exam is 60. While this is a closed book, closed notes exam, you are allowed to use your one page review sheet, front and back, written on an 8.5 x 11 inch physical piece of paper in your own handwriting. While your review sheet must have been created with 20th century technology, you may take this exam on a tablet if you like, or on paper.

Show all of your work (on the backs of pages, if necessary) and circle your answers. If you do not completely solve a problem, explain what you do understand about it. If you would like to guess about a problem, clearly say so. This will maximize your partial credit. No collaboration on this exam is allowed. Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total:	60	

---

1. (12 points) (a) Find the general solution to the differential equation

$$(4t^2 + t^4)\frac{dy}{dt} + 2t^3 y = 4t^3$$

(b) Find a solution where $y = 1$ when $t = 1$.

2. (12 points) (a) Find an integrating factor (function of x) for the differential equation

$$(3x^3y + x^2y^2) + (x^4 + x^3y)\frac{dy}{dx} = 0$$

that makes the differential equation exact.

(b) Find the general solution of the differential equation.

(c) Find the solution which has $y = 1$ when $x = 1$.

3. (12 points) (a) Find the general solution to the homogeneous differential equation

$$x^2y'' - 5xy' + 9y = 0 \quad , \; x > 0.$$

(b) Find a particular solution to the differential equation

$$x^2y'' - 5xy' + 9y = x^5 \quad , \; x > 0.$$

(c) Find the general solution to the differential equation

$$x^2y'' - 5xy' + 9y = x^5 \quad , \; x > 0.$$

4. (12 points) For this problem you may use the formulas in Table 6.2.1 that you copied into your review sheet, or work the problems directly. Compute the Laplace transform $F(s)$ of each of the following functions.

(a) $f(t) = e^{3t}$

(b) $f(t) = t^2 e^{3t}$

(c)

$$f(t) = \begin{cases} e^{3t} & \text{for } 0 \le t \le 4 \\ e^{4t} & \text{for } t > 4 \end{cases}$$

5. (12 points) A swinging door (that can open both to the inside and the outside) has both a spring and a dampener on it to control how well the door shuts. Define the angle of the swinging door at time t to be $\theta(t)$, where $\theta = 0$ corresponds to a shut door, $\theta > 0$ is opening to the outside, and $\theta < 0$ is opening to the inside.

   Suppose that the dynamics of the swinging door are controlled by

$$\theta''(t) = -9\theta(t) - 2k\theta'(t).$$

   In the above equation, the 4 represents the strength of the spring and the k represents the strength of the dampener (which resists motion in both directions).

   Find the general solution to the above differential equation when

   (a) k = 3

   (b) k = 4

   (c) k = 5

   (d) Of the three values of k listed above, which yields the most "optimally damped" swinging door, defined to be the case where the door may never hit the door jam at $\theta = 0$ (if we insert one) but otherwise has a **general solution** that closes as quickly as possible (meaning that for very large t, $\theta(t)$ is as small as possible)?